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9.Sc-Class-XII Relations and Functions Ex 1.1.

Q.1. Show that the relation R in the set $A = \{x \in Z : 0 \le x \le 12\}$ given by (i) $R = \{(a,b) : |a-b|$ is multiple of $4\}$

(ii) $R = \{(a,b) : a = b\}$, are equivalence relations. Find the set of all elements related to 1 in the each case.

Soln: It is given that $A = \{x \in Z : 0 \le x \le 12\}$
$= \{0,1,2,3,4,5,6,7,8,9,10,11,12\}$.

(I) $R = \{(a,b) : |a-b|$ is a multiple of $4\}$

For any element $a \in A$, we have $(a,a) \in R$ as $|a-a| = 0$ is a multiple of 4. Therefore, R is reflexive.

Now, let $(a,b) \in R$
$\Rightarrow |a-b|$ is a multiple of 4. $|b-a| = |-(a-b)| = |a-b|$ is multiple of 4. $\Rightarrow (b,a) \in R$. Therefore, R is symmetric. Now, let $(a,b), (b,c) \in R$.
So, $|a-b|$ and $|b-c|$ is multiple of 4.
$\Rightarrow |a-c|$ is also multiple of 4 $\Rightarrow (a,c) \in R$
Therefore, R is transitive.
Hence, R is an equivalence relation.
$\Rightarrow$ The set of elements related to 1 is $\{1,5,9\}$, since
$|1-1| = 0$ is multiple of 4.
$|5-1| = 4$ is a multiple of 4
$|9-1| = 8$ is a multiple of 4.

II. $R = \{(a,b) : a = b\}$.
For any element $a \in A$, we have $(a,a) \in R$, as $a = a$
$\therefore$ R is reflexive.
Now, let $(a,b) \in R \Rightarrow (b,a) \in R$
$\Rightarrow a = b \Rightarrow b = a$
$\Rightarrow (b,a) \in R$
$\therefore$ R is symmetric.
Now, Let $(a,b) \in R$ and $(b,c) \in R$
$\Rightarrow a = b$ and $b = c \Rightarrow (a,c) \in R$
$\therefore$ R is transitive.
Hence, R is an equivalence relation.
The element in R that are related to 1 will be those elements from set A. Which are equal to 1.
Hence, the set of elements related to 1 is $\{1\}$.

Q.2. Given example of relation which are symmetric but neither reflexive nor transitive.

Soln! Let $A = \{5, 6, 7\}$

We defined a relation R on A as $R = \{(5,6), (6,5)\}$

Relation R is not reflexive as $(5,5), (6,6), (7,7) \notin R$.

Now, as $(5,6) \in R$ and also $(6,5) \in R$. So, R is symmetric.

$(5,6), (6,5) \in R$ but $(5,5) \notin R$. Therefore, R is not transitive.

Hence, relation R is symmetric but neither reflexive nor transitive.

Q.3. Given example of relation which are transitive but neither reflexive nor symmetric.

Soln Let us consider a relation R in R defined as

$$R = \{(a,b) : a < b\}.$$

For any $a \in R$, We have $(a,a) \notin R$, since a cannot be strictly less than a itself. Therefore, R is not reflexive.

Now, $(1,2) \in R$ $[as\ 1 < 2]$.

But $(2,1) \notin R$ as 2 is not less than 1

$\therefore$ R is not symmetric

Now, let $(a,b), (b,c) \in R$

$\Rightarrow a < b$ and $b < c \Rightarrow a < c \Rightarrow (a,c) \in R$.

$\therefore$ R is transitive.

Hence, relation R is transitive but neither symmetric nor reflexive.

Q.4. Given example of relation which are reflexive and symmetric but not transitive.

Soln Let $A = \{4, 6, 8\}$

We defined a relation R on A as A

$= \{(4,4), (6,6), (8,8), (4,6), (6,4), (6,8), (8,6)\}$

Relation R is reflexive, since for every $a \in A$, $(a,a) \in R$ i.e. $(4,4), (6,6)$ $(8,8) \in R$.

Relation R is symmetric since $(a,b) \in R \Rightarrow (b,a) \in R$, $\forall a, b \in R$

Relation R is not transitive, since $(4,6), (6,8) \in R$, but $(4,8) \notin R$.

Hence, relation R is reflexive and symmetric but not transitive.